## Lesson 8.4: Part 1



A logarithm is just a special way to ask a specific question.


The Question:

| Exponential Form | Logarithmic Form |
| :---: | :---: |
|  |  |

# Oh no! <br> My logarithm has no base! 


$\log 10$
$\log \frac{1}{100}$
$\log 1000$

## The Logarithm Loop Trick

Always draw your loop counter-clockwise from the base!
$\log _{b} a=x \quad b^{x}=a$

## Example 1:

| Convert to exponential form. | Convert to logarithmic form. |
| :---: | :---: |
| $\log _{4} 16=2$ | $3^{2}=9$ |
| $\log _{5} 125=x$ | $2^{x}=18$ |

Example 2: Evaluate
$\log _{3} 81$

## Lesson 8.4: Part 2

## Special Values:

$\log$ of $1-\quad \log _{b} 1=0 \quad \rightarrow \quad b^{0}=1$
Log of base $\mathrm{b}-\quad \log _{b} b=1 \quad \rightarrow \quad b^{1}=1$

Common Logarithm: $\quad \log _{10} x=\log x \quad$ ** log key on calculator (base 10)

Natural Logarithm: $\quad \log _{e} x=\ln x \quad$ ** $\ln$ key on calculator (base e)

Inverse Logs: If $g(x)=\log _{b} x$ what is the inverse?
$g\left(g^{-1}(x)\right)=$
$g^{-1}(g(x))=$

Example 3: Find the inverse of the function.
a. $y=\log _{8} x$
b. $y=\ln (x-3)$

Inverse Properties:

$$
b^{\log _{b} x}=x \quad \log _{b} b^{x}=x
$$

Example 4: Use inverse properties to simplify.
a. $10^{\log x}$
b. $\log _{5} 125^{x}$

## Lesson 8.5: Properties of Logarithms

## Recall:

Exponent rules
a. Multiplication Property

$$
b^{m} \cdot b^{n}=b^{m+n}
$$

b. Division Exponents

$$
\frac{b^{m}}{b^{n}}=b^{m-n}
$$

| $\boldsymbol{l o g}_{\boldsymbol{b}} \boldsymbol{u}$ | $\boldsymbol{l o g}_{\boldsymbol{b}} \boldsymbol{v}$ | $\boldsymbol{\operatorname { l o g }}_{\boldsymbol{b}} \boldsymbol{u v}$ |
| :--- | :--- | :--- |
| $\log 10=$ | $\log 100=$ | $\log 1000=$ |
| $\log .1=$ | $\log .01=$ | $\log .001=$ |
| $\log _{2} 4=$ | $\log _{2} 8=$ | $\log _{2} 32=$ |

Conjecture? $\quad \log _{b} u v=$


Quotient Property: $\quad \log _{b} \frac{u}{v}=$

Power Property: $\quad \log _{b} u^{n}=$

## Example 1: Evaluate

$3^{x}=12$

Example 2: Expand
a. $\log _{10} 100 x^{6}$
b. $\log _{2} 9 x^{2}$

Example 3: Condense
a. $2 \log _{3} 7-5 \log _{3} x$
b. $\log _{4} 5+2 \log _{4} x$

Example 4: Given $\quad \log _{b} a=c \quad$ and $\quad \log _{b} d=e$ Evaluate the following
a. $\log _{b} \frac{a}{d}$
b. $\log _{b} a^{2}$
c. $\log _{b} a^{2} d^{3}$

Change of Base Formula
Let $\mathrm{x}, \mathrm{y}$, and b be positive numbers with $b \neq 0$ and $c \neq 1$

$$
\log _{x} y=\frac{\log _{b} y}{\log _{b} x} \quad \text { OR } \quad \log _{x} y=\frac{\ln y}{\ln x}
$$

Example 5: Use change of base formula to evaluate.

a. $\log _{6} 9$
b. $\log _{4} 8$

## Lesson 8.6: Solving Exponential and Logarithmic Equations

Properties:
If two powers with the same base are equal, then their exponents must be equal. If $b^{x}=b^{y}$, then $x=$ $y$ for $b>0 \& b \neq 1$

$$
\log _{b} x=\log _{b} y \quad \text { then } \quad x=y
$$

Example 1: $\quad 2^{4 x}=32^{x-1}$

$$
\text { Example 4: } \log _{4}(x+3)=\log _{4}(8 x+17)
$$

Example 2: $\quad 4^{x}=15$

Example 5: $\log _{4}(x+3)=2$

Example 3: $\quad 5^{x+2}+3=28$

## Recall:

The domain of logarithmic functions does not typically include all real numbers. Be sure to check your solutions for extraneous solutions.

Example 6: $\log _{2} x+\log _{2}(x-7)=3$

Example 7: $\log _{2}(2 x+2)=\log _{2}(x-7)$

