Name ____ **Date** ____________ **Hr** ___

Lesson 4.1: Matrix Operations

Matrix: rectangular arrangement of numbers in rows and columns

Naming a matrix: R x C

Equal Matrices: 2 matrices are equal if and only if they have the same dimensions and the entries in corresponding positions are equal.

Examples:

a. $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$ b. $\begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \end{bmatrix}$ c. $\begin{bmatrix} a & 1 & d \\ b & 2 & e \\ c & 3 & f \end{bmatrix}$

Example 1:
Are these matrices equal?

$$\begin{bmatrix} 5 & .5 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{25}{5} & \frac{1}{2} \\ -1 & \frac{6}{2} \end{bmatrix}$$

Example 2:
Add/Subtract Matrices ** Must have ________________________________

a. $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & -3 \\ 0 & 5 \end{bmatrix} =$

b. $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} - \begin{bmatrix} 5 & -3 \\ 0 & 5 \end{bmatrix} =$

Example 3:
Scalar Multiplication:

a. $5\begin{bmatrix}-1 & 0\\ 2 & 8\end{bmatrix} =$

b. $\frac{1}{2}\begin{bmatrix}-1 & 0\\ 2 & 8\end{bmatrix} =$

Example 4:
Solving Matrix Equations ** Must have ______________________________

a. $\begin{bmatrix}x & 7\\ 4 & 2y\end{bmatrix} = \begin{bmatrix}2 & 7\\ 4 & 8\end{bmatrix}$

b. $4\left(\begin{bmatrix}8 & 0\\ -1 & 2y\end{bmatrix} + \begin{bmatrix}4 & -2x\\ 1 & 6\end{bmatrix}\right) = \begin{bmatrix}48 & -48\\ 0 & 8\end{bmatrix}$

Example 5:
Given the following matrices, perform the following operations.

$$A = \begin{bmatrix} -1 & 2 \\ 6 & -10 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix}$$

a. A + B

b. 2(B – A)

c. $B - \frac{1}{2}A$

Lesson 4.2: Multiplying Matrices

Multiplying Matrices: If A is an M x N matrix and B is an N x P matrix, then the product is an M x P matrix

[M x N] x [N x P] = [M x P]

** The # of ___________ in the 1st matrix must match the # of _________ in the 2nd matrix to multiply.

Example 1:
Can the following matrices be multiplied?

a. [2 x 3] x [3 x 4]

b. [3 x 2] x [3 x 4]

Example 2:

$$\begin{bmatrix} -6 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -1 & 4 \\ -5 & 3 \end{bmatrix} =$$

Example 3:

$$\begin{bmatrix} 4 \\ 3 \\ 6 \end{bmatrix} \begin{bmatrix} 2 & 9 & 1 \end{bmatrix} =$$

Example 4:

$$\begin{bmatrix} 2 & 9 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 6 \end{bmatrix} =$$

Example 5:
Given the following matrices, perform the following operations.

$$A = \begin{bmatrix} -1 & 2 \\ 6 & -10 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix}$$

a. A(A + B)

b. AB

c. 3(AB)

Example 6:
The number of calories burned by people of different weights doing different activities for 20 minutes are shown in the matrix. Show how matrix multiplication can be used to write the total number of calories burned by 120 pound person and a 150 pound person who each bicycled for 40 minutes, jogged for 10 minutes, and then walked for 60 minutes.

Calories Burned

	120 lb.	**150 lb.**
Bicycling	109	136
Jogging	127	159
Walking	64	79

Lesson 4.3: Determinants and Cramers Rule

Part 1

Determinant: a real number associated with a **square** matrix; used to determine if a matrix has an inverse

Notation: [ ] is used around a matrix, | | is used to notate that the determinant is to be found

2 x 2 determinant: difference of the products of the entries on the diagonals.

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} =$$

Example 1:

Evaluate the determinant of the matrix.

$$\begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}$$

3 x 3 determinant

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} =$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

Example 2:

Given the following matrix, find the determinant.

$$\begin{bmatrix} 4 & 3 & 1 \\ 5 & -7 & 0 \\ 1 & -2 & 2 \end{bmatrix}$$

Example 3:

Find the area of a triangle with the given vertices.

$A(1,2),\ B(6,2),\ C(4,0)$

Part 2:

Cramers Rule: allows us to use determinants to solve a system of linear equations

**Named after Swiss mathematician Gabriel Cramer

Recall:

System of linear equations: two or more linear equations in the form:

$$\begin{cases} ax + by = e \\ cx + dy = f \end{cases}$$

Coefficient Matrix: the matrix formed only by the coefficients of x and y as they are found in standard form.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Cramers Rule: (2 x 2) Matrix

If $|A| \neq 0$, the system has exactly one solution.

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{detA} \qquad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{detA}$$

Example 1: Solve the system below using Cramers Rule

$$\begin{cases} 2x + y = 1 \\ 3x - 2y = -23 \end{cases}$$

<u>Cramers Rule</u>: (3 x 3) Matrix

Where $|A| \neq 0$, the system has exactly one solution and

$$\begin{cases} ax + by + cz = j \\ dx + ey + fz = k \\ gx + hy + iz = l \end{cases}$$

solution is:

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{detA} \qquad y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{detA} \qquad z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{detA}$$

$$detA = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

<u>Example 2</u>: Solve the system below using Cramers Rule

$$\begin{cases} 3x + 2y + 4z = 11 \\ 2x - y + 3z = 4 \\ 5x - 3y + 5z = -1 \end{cases}$$

Lesson 4.4: Identity and Inverse Matrices

Recall: Multiplicative identity

Identity Matrices:

2 x 2

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3 x 3

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If A is any n x n matrix and I is the identity matrix, then ____.

** The symbol for A inverse is A^{-1}

Example 1:

Given $A = \begin{bmatrix} 6 & 2 \\ 8 & 3 \end{bmatrix}$, what is A^{-1}?

Steps for finding A^{-1}:

1. Find the ____________________

2. Switch _____ and ______

3. Leave ___ and _____, but change their ________.

Check:

Inverse of a 2 x 2 Matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$

Example 2:
Find the inverse (A^{-1})

$$A = \begin{bmatrix} -6 & -7 \\ 2 & 2 \end{bmatrix}$$

Recall: $2x + 2 = 12$

Example 3:
Solve the matrix equation.

$$\begin{bmatrix} 5 & -1 \\ 8 & 2 \end{bmatrix} x = \begin{bmatrix} 17 & 20 \\ 26 & 20 \end{bmatrix}$$

** If you multiply both sides by _______________ then you will have x = answer (Note: Multiply to the ____________ on both sides, because matrices are NOT ________________________)

Lesson 4.4: Identity and Inverse Matrices (Calculator)

Use a graphing calculator to perform the indicated operations.

Example 1:
Find the determinant of the following matrix.

$$\begin{bmatrix} 6 & 2 \\ 8 & 3 \end{bmatrix}$$

Example 2:
Find the inverse (A^{-1})

$$A = \begin{bmatrix} -6 & -7 \\ 2 & 2 \end{bmatrix}$$

Check:

How do you know it there is no inverse?

Example 3:
Solve the matrix equation.

$$\begin{bmatrix} -5 & -3 \\ 4 & 1 \end{bmatrix} x = \begin{bmatrix} -12 & -5 & 18 \\ 4 & -3 & -13 \end{bmatrix}$$

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Section 4.5: Solving Systems Using Inverse Matrices

NOTES

I. A Matrix Equation

1. A **System of Equations** can be written as a matrix equation.

Example: Write a matrix equation for the system below.

$$\begin{cases} -3x + 4y = 5 \\ 2x - y = -10 \end{cases}$$

____

2. **The Solution of a Linear System:**

Let $AX = B$ represent a system of linear equations. If the determinant of A is nonzero,

then the linear system has exactly __________ solution, which is ______________________.

Example: Solve the system/matrix equation from above.

____

II. Solving Systems of Equations with Matrices and a Graphing Calculator

1. Write a Matrix Equation:

Write a matrix equation for the system below.

$$\begin{cases} x + 7y = 5 \\ -3x + 2y = 8 \end{cases}$$

____

2. Enter Matrix A and B into your calculator (see section 4.4 calculator directions, if needed).

$$A = \begin{bmatrix} 1 & 7 \\ -3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

3. To Solve the Matrix Equation and the System:

To solve the matrix equation $AX = B$, you need to multiply by the inverse of matrix A: $X = A^{-1}B$. Your calculator can do the multiplication for you.

Note: You must do the multiplication in this order, $A^{-1} \cdot B$, or it will not work.

What is the answer to the system at the top of this page? _______________

4. Example: Use a matrix equation to solve the system below.

$$\begin{cases} 4x - 2y + z = 3 \\ 2x - y + 3z = -6 \\ -2x + 3y - 2z = 1 \end{cases}$$

(a) Write the matrix equation for the system.

____

(b) Solve the matrix equation using your graphing calculator.

____

5. **Example:** Use a matrix equation to solve the system below.

$$\begin{cases} y = -2x + 5 \\ 4x + 2y = 6 \end{cases}$$

(a) Write the matrix equation for the system.

____

(b) Solve the matrix equation using your graphing calculator.

____