

Lesson 7.1: N^{th} Roots and Rational Exponents

Recall:

$$\sqrt{4} = 2 \quad \text{since}$$

Converting between Radical Form and
Rational Exponent Form

What is $\sqrt[3]{8}$?

$$\sqrt[\square]{x^{\square}} = x^{\frac{\square}{\square}}$$

Example 1: Rewrite the expression using rational exponent notation or radical notation. Evaluate

a. $\sqrt{n} =$

d. $\sqrt[n]{x} =$

b. $\sqrt[3]{n} =$

e. $4^{1/2} =$

c. $\sqrt[4]{81} =$

f. $8^{1/3} =$

Example 2: Evaluate without using a calculator. Show all work!

a. $\sqrt[3]{8} =$

f. $(4^{1/2})^3 =$

b. $\sqrt[3]{-8} =$

g. $\sqrt[n]{x^m} =$

c. $\sqrt[4]{16} =$

h. $8^{-2/3} =$

d. $\sqrt[4]{-16} =$

i. $16^{5/2} =$

e. $4^{3/2} =$

j. $64^{-2/3} =$

Example 3: Solve each equation.

a. $5y^4 = 80$

c. $\sqrt[5]{x+1} = 2$

b. $(x+1)^3 = 2$

d. $4(2x+4)^2 - 7 = 9$

7.2 Properties of Rational Exponents

Multiplication Property: keep the base and add the exponents.

Division Property: keep the base and subtract the exponents.

Power to a Power Property: keep the base and multiply the exponents.

Example 1: Simplify the expression.

a. $6^{\frac{1}{2}} \cdot 6^{\frac{1}{3}}$

c. $(4^3 \cdot 2^3)^{-\frac{1}{3}}$

b. $\left(27^{\frac{1}{3}} \cdot 6^{\frac{1}{4}}\right)^2$

d. $\frac{6}{6^{\frac{3}{4}}}$

e. $\left(\frac{18^{\frac{1}{4}}}{9^{\frac{1}{4}}}\right)^3$

Example 2: Write the expression in simplest form.

a. $\sqrt[4]{64}$

c. $\frac{\sqrt[3]{32}}{\sqrt[3]{4}}$

b. $\sqrt[3]{25} \cdot \sqrt[3]{5}$

d. $\sqrt[4]{\frac{7}{8}}$

Adding and Subtracting Roots and Radicals

Example 3: Perform the indicated operation.

a. $5\left(4^{\frac{3}{4}}\right) + 3\left(4^{\frac{3}{4}}\right)$

b. $\sqrt[3]{81} - \sqrt[3]{3}$

7.2 (Day 2 Notes)

The properties of rational exponents and radicals can also be applied to expressions involving variables. Because a variable can be either positive or negative, sometimes an absolute value is needed when simplifying a variable expression.

$$\sqrt[n]{x^n} = x \text{ when } n \text{ is an odd integer}$$

$$\sqrt[n]{x^n} = |x| \text{ when } n \text{ is an even integer}$$

NOTE: Absolute value is not needed when all the variables are assumed to be positive.

Example 1: Simplify the expression. Assume all variables are positive.

a. $\sqrt[3]{27a^9}$

d. $(16g^4h^2)^{1/2}$

b. $\sqrt[5]{\frac{x^5}{y^{10}}}$

e. $\frac{18rs^{2/3}t^4}{6r^{1/4}t^{-3}}$

c. $\sqrt[4]{32d^4e^9f^{14}}$

f. $\sqrt[5]{\frac{g^2}{h^7}}$

Example 2: Perform the indicated operation. Assume all variables are positive.

a. $8\sqrt{x} - 3\sqrt{x}$

b. $3gh^{1/4} - 6gh^{1/4}$

c. $2\sqrt[4]{6x^5} + x\sqrt[4]{6x}$