Lesson 6.3: Adding/Subtracting/Multiplying Polynomials

Examples:

a.
$$(5x^2 + x - 7) + (-3x^2 - 6x + 1)$$

b.
$$(5x^2 + x - 7) - (-3x^2 - 6x + 1)$$

c.
$$(3x^3 + 8x^2 - x - 5) - (5x^3 - x^2 + 17)$$

d.
$$(x+2)(5x^2+3x-1)$$

e.
$$3(5a+2)^2$$

f.
$$(2m-3)^3$$

Lesson 6.4: Factoring and Solving Polynomial Equations

Recall:

Factor:

a.
$$2x^2 - 5x - 3$$

b.
$$25x^2 - 64$$

c.
$$9x^2 + 15x$$

Difference of Two Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Sum of Two Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Examples

a.
$$x^3 + 8 =$$

b.
$$x^3 - 8 =$$

c.
$$64a^4 - 27a =$$

Factoring by Grouping

$$x^2y^2 - 3x^2 - 4y^2 + 12$$

Examples:

a.
$$4a^6 - 4b^6$$

b.
$$a^2b^2 - 8ab^3 + 16b^4$$

c.
$$42x^2 - 24xy - 18y^2$$

Factor and Solve. (Find all real and imaginary solutions)

a.
$$2x^5 - 18x = 0$$

b.
$$2x^4 - 5x^2 - 3 = 0$$

Lesson 6.5: Remainder and Factor Theorems

Polynomial Long Division

$$\frac{y^4 + 2y^2 - y + 5}{y^2 - y + 1}$$

Example: $\frac{x^3 - x^2 - 2x + 8}{x - 1}$

Long Division: Synthetic Division:

Factor Theorem:

A polynomial f(x) has a factor y - k iff f(k) = 0

Example: Factor $f(x) = 3x^3 + 13x^2 + 2x - 8$ given f(-4) = 0

Example: If one zero of $f(x) = x^3 + 6x^2 + 3x - 10$ is x = -5, find the other zeros of the function.

Lesson 6.6: Finding Rational Zeros

Given
$$f(x) = 3x^3 + 13x^2 + 2x - 8$$
 factor if $f(-4) = 0$
 $f(x) = (x + 4)(x + 1)(3x - 2)$

zeros:
$$x = -4, -1, \frac{2}{3}$$

- ** Notice the numerators (-4, -1, 2) are factors of the ______.
- ** The denominators (1, 3) are factors of the ______.

Rational Zero Theorem: If $f(x) = ax^n + bx^{n-1} + \cdots + k$ has integer coefficients, then every rational zero of f has the following form.

$$\pm \frac{p}{q} = \frac{\text{factors of the constant term } k}{\text{factors of the leading coefficient } a}$$

Example 1: Find the rational zeros of $f(x) = x^3 - 4x^2 - 11x + 30$

$$p =$$

$$q =$$

Possible rational zeros ±



Example 3: Find all real zeros (rational and irrational) of $3x^4 + 11x^3 + 11x^2 + x - 2$

Lesson 6.7: The Fundamental Theorem of Algebra

Recall:

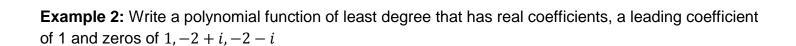
**Counting all real and imaginary solutions and repeating roots

Example 1: Find the number of solutions and write them.

a.
$$x^3 + 3x^2 + 16x + 48 = 0$$

b.
$$x^2 - 14x + 49 = 0$$

c.
$$x^4 + 3x^3 - 8x^2 - 22x - 24 = 0$$



Example 3: Approximate the zeros using your calculator. How many total solutions are there? Are they real or imaginary solutions?

a.
$$f(x) = x^3 - 4x^2 - 5x + 14$$

b.
$$f(x) = x^4 - 2x^3 - x^2 - 2x - 2$$