Lesson 6.3: Adding/Subtracting/Multiplying Polynomials

Examples:

a. $(5x^2 + x - 7) + (-3x^2 - 6x + 1)$

b. $(5x^2 + x - 7) - (-3x^2 - 6x + 1)$

c. $(3x^3 + 8x^2 - x - 5) - (5x^3 - x^2 + 17)$

d. $(x + 2)(5x^2 + 3x - 1)$

e. $3(5a + 2)^2$

f. $(2m - 3)^3$

Lesson 6.4: Factoring and Solving Polynomial Equations

Recall:

Factor:

a. $2x^2 - 5x - 3$

b. $25x^2 - 64$

c. $9x^2 + 15x$

Difference of Two Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Sum of Two Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Examples

a. $x^3 + 8 =$

b. $x^3 - 8 =$

c. $64a^4 - 27a =$

Factoring by Grouping

$x^2y^2 - 3x^2 - 4y^2 + 12$

Examples:

a. $4a^6 - 4b^6$

b. $a^2b^2 - 8ab^3 + 16b^4$

c. $42x^2 - 24xy - 18y^2$

Factor and Solve. (Find all real and imaginary solutions)

a. $2x^5 - 18x = 0$

b. $2x^4 - 5x^2 - 3 = 0$

Lesson 6.5: Remainder and Factor Theorems

Polynomial Long Division

$$\frac{y^4+2y^2-y+5}{y^2-y+1}$$

Example: $\frac{x^3-x^2-2x+8}{x-1}$

Long Division:

Synthetic Division:

Factor Theorem:

A polynomial $f(x)$ has a factor $y - k$ iff $f(k) = 0$

Example: Factor $f(x) = 3x^3 + 13x^2 + 2x - 8$ given $f(-4) = 0$

Example: If one zero of $f(x) = x^3 + 6x^2 + 3x - 10$ is $x = -5$, find the other zeros of the function.

Lesson 6.6: Finding Rational Zeros

Given $f(x) = 3x^3 + 13x^2 + 2x - 8$ factor if $f(-4) = 0$

$f(x) = (x + 4)(x + 1)(3x - 2)$

zeros: $x = -4, -1, \frac{2}{3}$

** Notice the numerators (-4, -1, 2) are factors of the _____.

** The denominators (1, 3) are factors of the _____.

Rational Zero Theorem: If $f(x) = ax^n + bx^{n-1} + \cdots + k$ has integer coefficients, then every rational zero of f has the following form.

$$\pm\frac{p}{q} = \frac{\text{factors of the constant term } k}{\text{factors of the leading coefficient } a}$$

Example 1: Find the rational zeros of $f(x) = x^3 - 4x^2 - 11x + 30$

$p =$

$q =$

Possible rational zeros $\pm$

Example 2: Find all real zeros (rational and irrational) of $f(x) = x^3 - 7x^2 + 10x + 6$

Example 3: Find all real zeros (rational and irrational) of $3x^4 + 11x^3 + 11x^2 + x - 2$

Lesson 6.7: The Fundamental Theorem of Algebra

Recall:

1st degree – 1 solution

2nd degree – 2 solutions

3rd degree – 3 solutions

**Counting all real and imaginary solutions and repeating roots

Example 1: Find the number of solutions and write them.

a. $x^3 + 3x^2 + 16x + 48 = 0$

b. $x^2 - 14x + 49 = 0$

c. $x^4 + 3x^3 - 8x^2 - 22x - 24 = 0$

Example 2: Write a polynomial function of least degree that has real coefficients, a leading coefficient of 1 and zeros of $1, -2+i, -2-i$

Example 3: Approximate the zeros using your calculator. How many total solutions are there? Are they real or imaginary solutions?

a. $f(x) = x^3 - 4x^2 - 5x + 14$

b. $f(x) = x^4 - 2x^3 - x^2 - 2x - 2$